Statistics of Transverse Velocity Differences in Turbulence

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Abstract. An unusual symmetry of the equation for the generating function of transverse velocity differences $\Delta v = v(x+r) - v(x)$ is used to obtain a closed equation for the probability density function $P(\Delta v, r)$ in strong three-dimensional turbulence. It is shown that the terms, mixing longitudinal and transverse components of velocity field, dominate the pressure contributions. The dissipation terms are closed on qualitative grounds. The resulting equation gives the shape of the pdf, anomalous scaling exponents and amplitudes of the moments of transverse velocity differences.

Intermittency of strong turbulence seems to be a well-established experimental fact. Manifested as anomalous scaling of velocity structure functions, this phenomenon resisted a theoretical description for almost three decades (for an extensive review see [1]-[3]). Early attempts to attack the problem (cascade models) were based on the so called refined similarity hypothesis (RSH), connecting dissipation fluctuations with those of the differences of longitudinal components of velocity field. Only recently the properties of tranverse components of velocity field gained some interest [4]-[5]. In this paper we explore a remarkable feature of the equations for the generating function of tranverse components of velocity field discovered in [6] to derive the equation for the probability density (pdf) of transverse velocity differences. The solution to this equation describes the experimentally observed shape of the pdf, anomalous scaling exponents and the amplitudes of the structure functions.

We are interested in statistical properties of an incompressible fluid strirred by a random force in the right side of the Navier-Stokes equations, defined by the pair correlation function:

$$< f_i(\mathbf{k}) f_j(\mathbf{k}') > \propto P(\delta_{ij} - \frac{k_i k_j}{k^2}) \frac{\delta(k-k_f)}{k^{d-1}} \delta(\mathbf{k}+\mathbf{k}')\delta(t-t') \tag{1}$$

so that $(f(x+r)-f(r))^2 \propto P(1-cos(k_f r))$. The integral scale of the problem $L \approx 1/k_f$ and d stands for space dimensionality. Consider two points $\mathbf{x}$ and $\mathbf{x}'$ and define $\mathbf{r} = \mathbf{x} - \mathbf{x}'$. Assuming that the x-axis is parallel to the displacement vector $\mathbf{r}$, one can find for the separation r in the inertial range ([1], [7], [8]) that $S_3 =< (\Delta u)^3 > \propto Pr$ and $S_3^t = \overline{(\Delta v)^3} \equiv \overline{(v(x')-v(x))^3} = 0$ where u and v are components of velocity field parallel and perpendicular to the x-axis (vector $\mathbf{r}$), respectively. These relations , resulting from the Navier-Stokes equations, are dynamic constraints on any theory of turbulence. The pumping power $P = O(1)$ is a constant. In what follows we will often set $P = 1$, $L = 1$ and restore the correct dimensionality at the end of calculations. We also have [1], [7], [8]: $r^{2-d}\partial_r(r^{d-1}S_2) = (d-1)S_2^t \equiv (d-1)\overline{(\Delta v)^2}$ and for $d = 3$:

$$6S_{3t} \equiv 6\overline{\Delta u(\Delta v)^2} = \partial_r r S_3 \propto Pr \tag{2}$$

which will be important in what follows.

Following [9], [6] and [10] we consider the N-point generating function:

$$Z =< e^{\lambda_i \cdot \mathbf{v}(\mathbf{x_i})} > \tag{3}$$

where the vectors $\mathbf{x_i}$ define the positions of the points denoted by $1 \le i \le N$ and summation over $x_j \; x_j$ is assumed. Using the Navier-Stokes equations and incompressibility condition, the equation for $\partial_t Z = \lambda_{i\mu} < (\partial_t v_\mu(x_i))exp(\lambda \cdot \mathbf{v}) >$ can be written [6], [10]:

$$\frac{\partial Z}{\partial t} + \frac{\partial^2 Z}{\partial \lambda_{i,\mu} \partial x_{i,\mu}} = I_f + I_p + D \tag{4}$$

with the forcing, pressure and dissipation contributions I_f; I_p; D defined below. In what follows we will be mainly interested in the probability density function of the two-point velocity differences which is obtained from (3)-(4), setting $\lambda_\mathbf{1} + \lambda_\mathbf{2} = \mathbf{0}$ (see Refs. [6], [9]-[10]), so that $Z =< exp(\lambda \cdot \mathbf{U}) >$, where $\mathbf{U} = \mathbf{u}(\mathbf{x}') - \mathbf{u}(\mathbf{x}) \equiv \Delta \mathbf{u}$. Assuming , as in [9]-[10], that if $|U| << u_{rms}$ and $r << L$, the inertial range variable Δu is independent of the "large-scale" variable $U_+ = u(x) + u(x+r)$ gives:

$$\frac{\partial Z}{\partial t} + \frac{\partial^2 Z}{\partial \lambda_\mu \partial r_\mu} = I_f + I_p + D \tag{5}$$

In a statistically isotropic and homogeneous flow the generating function can depend only on three variables:

$$\eta_1 = r; \quad \eta_2 = \frac{\lambda \cdot \mathbf{r}}{r} \equiv \lambda cos(\theta); \quad \eta_3 = \sqrt{\lambda^2 - \eta_2^2};$$

In these variables ($\eta_1 = r > 0$):

$$Z_t + [\partial_{\eta_1} \partial_{\eta_2} + \frac{d-1}{r} \partial_{\eta_2} + \frac{\eta_3}{r} \partial_{\eta_2} \partial_{\eta_3} + \frac{(2-d)\eta_2}{r\eta_3} \partial_{\eta_3} - \frac{\eta_2}{r} \partial^2_{\eta_3}] Z = I_f + I_p + D \tag{6}$$

where

$$I_p = \lambda_i < (\partial_{2,i} p(2) - \partial_{1,i} p(1)) e^{\lambda \cdot \mathbf{U}} > \tag{7}$$

$$I_f = (\eta_2^2 + \eta_3^2) P (1 - cos(k_f r)) Z \tag{8}$$

$$D = \nu \lambda_{i\mu} < (\partial^2_{2,j} v_{i\mu}(2) - \partial^2_{1,j} v_{i\mu}(1)) e^{\lambda \cdot \mathbf{U}} >= -(\eta_2^2 + \eta_3^2) < (\mathcal{E}(2) + \mathcal{E}(1)) e^{\eta_\mathbf{2} \mathbf{\Delta u} + \eta_\mathbf{3} \mathbf{\Delta v}} > \tag{9}$$

where, to simplify notation we set $\partial_{i,\alpha} \equiv \frac{\partial}{\partial x.\alpha}$ and $v(i) \equiv v(\mathbf{x_i})$ and $\lambda_x = \eta_2$ and $\lambda_y = \eta_3$. The dissipation rate $\mathcal{E}(x) = \nu(\partial_i v_j)^2$. The generating function, depending on a single angle θ, can be written as:

$$Z =< e^{\eta_2 \Delta u + \sqrt{d-1}\eta_3 \Delta v} > \tag{10}$$

so that any correlation function

$$< (\Delta u)^n (\Delta v)^m >= (d-1)^{-\frac{m}{2}} \frac{\partial^n}{\partial \eta_2^n} \frac{\partial^m}{\partial \eta_3^m} Z(\eta_2 = \eta_3 = 0) \tag{11}$$

Differentiating the equation (6) and setting both $\eta_2 = \eta_3 = 0$ gives all known kinematic and dynamic constraints, outlined above. It is clear from the symmetries of the problem that the transverse velocity difference probability density is symmetric, i.e. $P(\Delta v, r) = P(-\Delta v, r)$. We are interested in the equation (6)-(9) in the limit $\eta_2 \rightarrow 0$. Let us first discuss some of the general properties of incompressible turbulence. Consider the "one-component forcing fucntion" $\mathbf{f}(x,y,z) = (f_x(x,y,z), 0, 0)$ satisfying the incompressibility constraint $(\nabla \cdot \mathbf{f} = 0)$. In this case the equation (6) is:

$$[\partial_{\eta_1}\partial_{\eta_2} + \frac{2}{r}\partial_{\eta_2} + \frac{\eta_3}{r}\frac{\partial^2}{\partial_{\eta_2}\partial\eta_3} - \frac{\eta_2}{r}\frac{\partial^2}{\partial{\eta_3}^2} - \eta_2^2(1 - cos(k_f r))]Z = I_p + D \tag{12}$$

Then, setting $\eta_2 = 0$ removes all information about the forcing function from the equation of motion. The equation (12) explicitely assumes that the flow at the scale $r << L$ is statistically isotropic and homogeneous This can happen due to the pressure terms $\Delta p = -\nabla_i \nabla_j v_i v_j$, effectively mixing various components of the velocity field. This universality, i.e. independence of the small-scale turbulence on the symmetries of the large-scale forcing is an assumption of this work.

One property of equation (12) deserves discussion [6]. Neglecting for a time being I_p and D we have in the limit $\eta_2 \rightarrow 0$

$$[\partial_{\eta_1}\partial_{\eta_2} + \frac{2}{r}\partial_{\eta_2} + \frac{\eta_3}{r}\frac{\partial^2}{\partial_{\eta_2}\partial\eta_3}]Z(\eta_2 = 0, \eta_3, r) = 0 \tag{13}$$

and

$$[\partial_r + \frac{2}{r} + \frac{\eta_3}{r}\frac{\partial}{\partial\eta_3}]Z(0,\eta_3,r) = \Psi(\eta_3) \tag{14}$$

where an arbitrary function $\Psi(\eta_3)$ can be chosen to satisfy various dynamic constraints. The fact that equation (14) contains only one derivative means that the underlying dynamic equation is linear, provided both I_p and D contributions involve ∂_{η_2} and the total order of the original equation $n \leq 2$. It will become clear below that this is the case in two-dimensional turbulence, while the situation in 3D is more complex due to dissipation contributions, absent in 2D.

First, we will evaluate the pressure contribution I_p which when $\eta_2 \to 0$, can be rewritten as:

$$I_p \approx \eta_3 < (\partial_y p(0) - \partial_{y'} p(r))exp(\sqrt{d-1}\eta_3\Delta v + \eta_2\Delta u) > \tag{15}$$

$$\partial_y p(0) - \partial_{y'} p(r) = \int k_y(1-e^{ik_x r})[\frac{k_x^2}{k^2}u(q)u(k-q) + \frac{k_y^2}{k^2}v(q)v(k-q) + \frac{k_x k_y}{k^2}u(q)v(k-q)]d^2k d^d q \tag{16}$$

and the exponent is expressed simply as:

$$e^{\sqrt{d-1}\eta_3\Delta v + \eta_2\Delta u} = exp[\sqrt{d-1}\eta_3 \int (1-e^{iQ_x r})v(Q)d^2Q + \eta_2 \int (1-e^{iQ_x r})u(Q)d^2Q] \tag{17}$$

The k_y-integration is carried out over the interval $(-\infty,\infty)$, so that only even powers of k_y can contribute to the integral. The most interesting feature of expressions (15)-(17) is that the additional k_y-factors can appear only from various correlation functions resulting from the expansion of the exponents. This is a consequence of the fact that here we are interested in the y-components of the pressure gradients which are perpendicular to the displacement vector $\mathbf{r}$.

Let us consider the $N = n + m$-rank tensor

$$T_N =< v_{i1}(x)v_{j2}(x)\cdots v_{an}(x)v_{\alpha,n+1}(x+r)\cdots v_{\omega,n+m}(x+r) > \tag{18}$$

In a statistically isotropic flow this tensor can be expressed only in terms of Kronekker symbols δ_{ij}, componnets of the displacement vector r_α and some functions $A(|\mathbf{r}|)$. Thus the

tensor can be represented as a sum of terms having a general structure $A(r)\delta_{ij}\cdots\delta_{pc}r_j\cdots r_\alpha$. The product of h Kronekkers contains an even $(2h)$ number of indices, while the product of $N-2h$ components of the displacement vector are the ones we are interested in. We have

$$T_N = \int < v_i(q_1)v_j(q_2)\cdots v_a(q_n)v_\alpha(q_{n+1})\cdots v_\omega(q_{n+m}) > e^{i\mathbf{Q}\cdot\mathbf{r}} \tag{19}$$

where $\mathbf{Q} = \mathbf{q_{n+1}} + \mathbf{q_{n+2}}\cdots\mathbf{q_{n+m}}$. The tensor T_N can be represented as a sum of contributions having the following structure: $t_{2h}\int A(Q)\partial_{Q_j}\cdots\partial_{Q_\alpha}e^{i\mathbf{Q}\cdot\mathbf{r}}$, where t_{2h} is a product of the h Kronekker symbols. As a consequence, a typical contribution to the correlation function $<(v_i(q_1)v_j(q_2)\cdots v_a(q_n)v_\alpha(q_{n+1})\cdots v_\omega(q_{n+m})>$ can be written as: $t_{2h}B(|Q|)Q_j\cdots Q_\alpha$ where $B(Q)\propto A(Q)/Q^m$. Now we consider each term in the right side of (16). In the limit $\eta_2\rightarrow 0$ the expansion of the exponents gives only various powers of $v(q)$'s. The first $O(u^2)$ term generates two kinds of correlation functions

$$< uuv^{2n} > \propto k_x^2 k_y^{2n} \tag{20}$$

and $< uuv^{2n+1} >= 0$. The second term in (16) generates a typical contribution

$$< vvv^m > \propto k_y^m \tag{21}$$

if m is even- and is equal to zero if m is an odd number. Thus, both first and second terms in the right side of (16) are equal to zero. We are left with the third contribution, mixing the u and v-componnets of the velocity field. This means that, neglecting possible infra-red divergences the estimate for the remaining term gives

$$I_p = b\frac{\eta_3}{r} < \Delta u\Delta v e^{\eta_2\Delta u+\eta_3\Delta v} >= b\frac{\eta_3}{r}\frac{\partial^2}{\partial\eta_2\partial\eta_3}Z(\eta_2=0,\eta_3,r) \tag{22}$$

This result was first obtained for a close- to- gaussian case of two-dimensional flow [6]. Using some additional assumptions about the structure of the $\Psi(\eta_3)$ the gaussian expression for the PDF of transverse velocity differences in two-dimensional turbulence was obtained. Here we have proved that, in general, the only contribution to the I_p - term comes from the one mixing u and v components of velocity field.

The difficulty of the three-dimensional case comes from the dissipation contributions. This is easily understood on the basis of the Navier-Stokes equations. Introduce the local value of the kinetic energy $K = v^2/2$ and $Q = K(x+r) + K(r)$. It is clear that

$$\mathcal{E}(x+r)+\mathcal{E}(x) \approx \partial_t Q+\partial_{2,i}v_i(x+r)K(x+r)+\partial_{1,i}v_i(x)K(x)+\partial_{2,i}v_i(x+r)p(x+r)+\partial_{1,i}v_i(x)p(x) \tag{23}$$

where the $O(\nu)$ terms are not written down. One can see that the $O(\nabla v^3)$ dissipation contributions are much more complex than the $O(\nabla v^2)$ pressure terms, derived above. Thus,

$$< (\mathcal{E}(2)+\mathcal{E}(1))e^{\lambda\cdot\mathbf{\Delta v}} >\approx$$

$$\frac{\partial}{\partial r_i} < (v_i(2)K(2)-v_i(1)K(1))e^{\lambda\cdot\mathbf{\Delta v}} > -\lambda_j < (v_i(2)K(2)s_{ij}(2)-v_i(1)K(1)s_{ij}(1))e^{\lambda\cdot\mathbf{\Delta v}} > +R$$

where $s_{ij}(m) = \frac{\partial v_j(m)}{\partial x_i}$ and R denotes the remaining contributions coming from (23) and the viscous terms. Assuming that the integral scale L can enter the equations through the velocity differences only we make a proposition: in the limit $\eta_2 \to 0$:

$$< (\mathcal{E}(2)+\mathcal{E}(1))e^{\lambda\mathbf{\Delta v}} >\approx< \Delta u\Delta v\frac{\partial\Delta v}{\partial r}e^{\lambda\cdot\mathbf{\Delta v}} >\approx< \Delta u\Delta v\frac{\partial\Delta v}{\partial r}e^{\eta_2\Delta u+\sqrt{d-1}\eta_3\Delta v} > \tag{24}$$

Assumption (24) is essential for the present theory. In its spirit it resembles Kolmogorov's refined similarity hypothesis, connecting the dissipation rate, averaged over a region of radius r, with $(\Delta u)^3$. It will become clear below that in the limit $\eta_2 \to 0$ the dimensionally correct expression

$$D^o \approx \eta_2^2\frac{\partial}{\partial r_i} < \Delta v_i(\Delta v_j)^2 e^{\eta_2\Delta u+\sqrt{d-1}\eta_3\Delta v} >\propto \eta_2^2\partial_r\partial_{\eta_2}\partial_{\eta_3}^2 Z$$

cannot appear in the equation for Z, since it leads to exponents of the structure functions violating the Holder inequality. It means that the model (24) can be valid only as a result of mutual compensation of various contributions to the expression for D defined by (9) and (23). At the present time we cannot prove that it is so. With the assumption (24) we have:

$$D \propto \eta_3 \partial_{\eta_2} \partial_{\eta_3} \partial_r Z \tag{25}$$

Thus, in the limit $\eta_2 \to 0$:

$$[\partial_{\eta_1} \partial_{\eta_2} + \frac{2}{r} \partial_{\eta_2} + \frac{\eta_3}{r} \frac{\partial^2}{\partial_{\eta_2} \partial \eta_3} - \frac{\eta_2}{r} \frac{\partial^2}{\partial {\eta_3}^2} + b \frac{\eta_3}{r} \frac{\partial^2}{\partial \eta_2 \partial \eta_3} + c \eta_3 \partial_{\eta_2} \partial_{\eta_3} \partial_r] Z(\eta_2 = 0, \eta_3, r) = 0 \tag{26}$$

where unknown coefficients b and c will be determined below. Integrating (26) over η_2 gives

$$[\partial_{\eta_1} + \frac{2}{r} + \frac{\eta_3}{r} \frac{\partial}{\partial \eta_3} + b \frac{\eta_3}{r} \frac{\partial}{\partial \eta_3} + c \eta_3 \partial_{\eta_3} \partial_r] Z(\eta_2 = 0, \eta_3, r) = \Psi(\eta_3, r) \tag{27}$$

We must choose $\Psi(\eta_3)$ in such a way that the generating function $Z(0,0,r) = 1$. Inverse Laplace transform gives the resulting equation for the pdf $P(\Delta v, r)$

$$\frac{\partial P}{\partial r} + \frac{1+3\beta}{3r} \frac{\partial}{\partial V} V P - \beta \frac{\partial}{\partial V} V \frac{\partial P}{\partial r} = 0 \tag{28}$$

Since $S_3^t = 0$, the coefficients in the equation for the pdf $P(V,r) = P(-V,r)$ (28) are chosen to give $s_3^t = \overline{|\Delta v|^3} = a^3 Pr$ with an undetermined amplitude a^3. This is an assumption of the present theory, not based on rigorous theoretical considerations. Seeking the solution in a form $S_n^t =< (\Delta v)^n > \propto r^{\xi_n}$ we obtain

$$\xi_n = \frac{1+3\beta}{3(1+\beta n)} n \approx \frac{1.15}{3(1+0.05n)} n \tag{29}$$

which was derived in [10] together with $\beta \approx 0.05$. The equation (29) gives: $P(0,r) \propto r^{-\kappa}$ where $\kappa = \frac{1+3\beta}{3(1-\beta)} \approx 0.4$ for $\beta = 0.05$. Very often the experimental data are presented as $P(X,r)$ where $X = V/r^{\mu}$ with $2\mu = \xi_2 \approx 0.696$ for $\beta = 0.05$. This gives

$$P(X=0,r) \propto r^{-\kappa+\mu} \approx r^{-0.052} \tag{30}$$

The experimental data by Sreenivassan [11] give $-\kappa + \mu \approx -0.06$. Introducing $P(V,r) = r^{-\kappa} F(V,r)$ leads to:

$$(1-\beta) r \frac{\partial F}{\partial r} + \kappa V \frac{\partial F}{\partial V} - \beta V r \frac{\partial^2 F}{\partial V \partial r} = 0 \tag{31}$$

Next, we define $Y = V/r^{\kappa}$ so that:

$$(1-\beta)r\frac{\partial F}{\partial r} + 2\beta\kappa Y\frac{\partial F}{\partial Y} + \beta\kappa Y^2\frac{\partial^2 F}{\partial Y^2} - \beta\kappa Y r\frac{\partial^2 F}{\partial Y \partial r} = 0 \tag{32}$$

Defining $-\infty < y = Ln(Y) < \infty$, substituting this variable into (32) and evaluating the Fourier transform of the resulting equation gives:

$$(1-\beta)r\frac{\partial F}{\partial r} + \beta\kappa(ik - k^2)F - ik\beta r\frac{\partial F}{\partial r} = 0 \tag{33}$$

with the result: $F \propto r^{\gamma(k)}$, where

$$\gamma(k) = \beta\kappa\frac{-ik + k^2}{1 - \beta - i\beta k}Ln(r/L) \tag{34}$$

with $r/L << 1$. We have to evaluate the inverse Fourier transform:

$$F = \int_{-\infty}^{\infty} dk e^{-iky} e^{\gamma(k)} \tag{35}$$

in the limit $y = O(1)$ and $r \to 0$ so that $Ln(r/L) \to -\infty$. The integral can be evaluated eaxctly. However, the resulting expression is very involved. Expanding the denominator in (35) gives :

$$F = \int_{-\infty}^{\infty} dk e^{-ik(y + \frac{\beta\kappa|Ln(r)|}{1-\beta})} e^{-\frac{\beta\kappa(1+\beta)|Ln(r)|}{(1-\beta)}k^2} \tag{36}$$

and

$$F \propto \frac{1}{\sqrt{\Omega(r)}} exp(-\frac{(Ln(\xi))^2}{4\Omega}) \tag{37}$$

with $\xi = V/r^{\frac{\kappa}{1-\beta}}$ and $\Omega(r) = 4\beta\kappa\frac{1+\beta}{1-\beta}|Ln(r/L)|$.

To understand the range of validity of this expression, let us evaluate $< V^n >$ using the expression (37) for the pdf. Simple integration, neglecting $O(\beta^2)$ contributions, gives: $< V^n > \propto r^{\alpha_n}$ with $\alpha_n = (1 + 3\beta)(n - \beta(n^2 + 2))/3$. Comparing this relation with the exact result (30) we conclude that the expression for the pdf, calculated above, is valid in the range $n >> 1$ and $\beta n << 1$. The properties of the pdf in the range $3 \leq \xi \leq 15$ are demonstrated

on Figs. 1 and Fig.2 for $r/L = 0.1;\ 0.01;\ 0.001$. Fig.2 shows the dependence of the pdf on r for two values of $\xi = 10;\ 15$. In this range of parameter variation the curves are reasonably well approximated by the the function

$$P(\xi, r) \propto e^{-a(r)\xi^{0.9}} \tag{38}$$

wwith $a(r) \approx r^{\theta}$ wwith $\theta \approx 0.12 - 0.15$. All quantitative predictions (29), (30) and (38) can be easily veryfied exprerimentally. Indirect conformation of this result can be found in the experimental data by Gagne [12] on the asymmetric pdf of longitudinal velocity differences: the curves were approximated by an exponential with width $a(r)(r) \propto r^{\theta^o}$ with $\theta^o \approx 0.15$.

The log-normal distribution (37), derived from (26), (28), is valid in a certain (wide but limited) range of V- variation. It is clear from (26)-(28) that neglecting the dissipation terms ($c \propto \beta = 0$) leads to $\xi_n = n/3$, i. e. disappearence of anomalous scaling of moments of velocity differences. This result agrees with the well-developed phenomenology, attributing intermittency to the dissipation rate fluctuations: the stronger the fluctuations, the smaller fraction of the total space they occupy [1], [8]. To the best of our knowledge, this is the first work leading to multifractal distribution of velocity differences as a result of approximations made directly on the Navier-Stokes equations. The expression (37) is similar to the one obtained in the groundbreaking paper by Polyakov on the scale-invariance of strong interactions, where the multifractal scaling and the pdf were analytically derived for the first time [13]-[14]. In the review paper [14] Polyakov noticed that the exact result can be simply reproduced considering a cascade process with a heavy stream (particle) transformed into lighter streams at each step of the cascade (fission). Due to relativistic effects the higher the energy of the particle, the smaller the angle of a cone, accessible to the fragments formed as a result of fission. Thus, the larger the number of a cascade step, the smaller is the fraction of space occupied by the particles [14]. It is remarkable that the qualitatively similar picture was so successfully applied by Parisi and Frisch to the undertanding of scale invariance of strong turbulence [15].

To conclude: theory of turbulence needs a closure for the pressure gradient and dissipation terms in the equations for the probability density. The task is greatly simplified by an

unusual symmetry of the equations for the pdf of transverse velocity differences, resulting in reduction of the order of the equation for the generating function Z. It means that, not being able to evaluate the entire probability density $P(\Delta u, \Delta v, r)$, we can derive an expression for $P(\Delta v, r) = \int_{-\infty}^{\infty} P(\Delta u, \Delta v, r) d\Delta u$. It has been shown in this work that the dominant contributions to the expression for the pressure gradients comes from the terms, "mixing" transverse and longitudinal components of the velocity field. The self-interaction does not affect the dynamics of transverse velocity differences. We believe, the expression (22) stands on a relatively firm ground. The closure of the dissipation terms (24), (25) is an assumption which must be tested experimentally. The proposed closure resembles the kolmogorov refined similarity hypothesis connecting the dissipation rate averaged over a "ball" of a radius r around point x with the third power of velocity difference $(\Delta u)^3$. The relation (24)-(25), though, is based on an equivalent mixed third-order structure function (2), untill now overlooked by studies of strong turbulence.

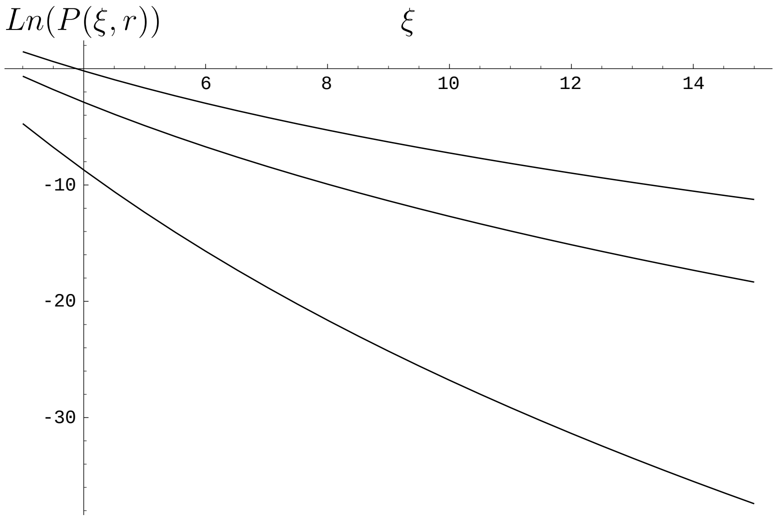

Fig.1. $Ln(P(\xi, r))$. From bottom to top: r/L=0.1; 0.01; 0.001, respectively.

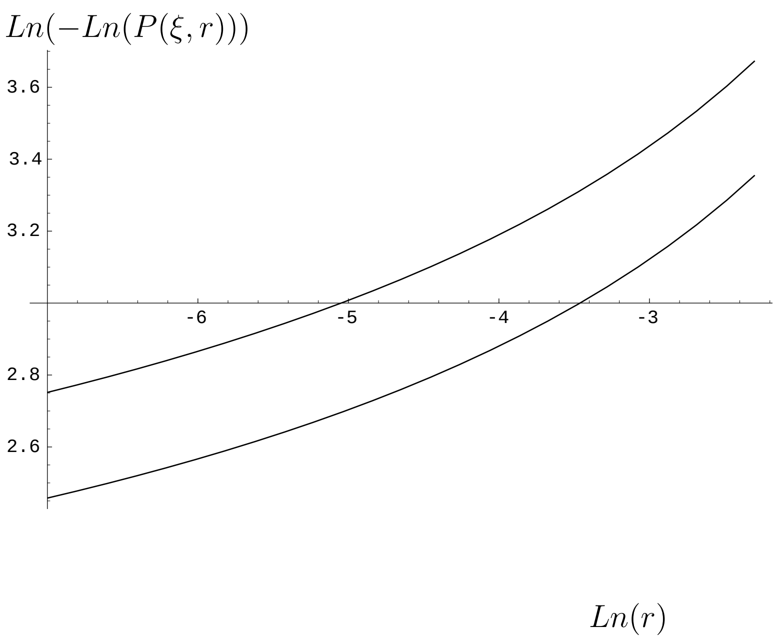

Fig.2. $Ln(-Ln(P(\xi, r)))$ vs. $Ln(r)$ for $\xi = 10$ (top) and $\xi = 15$.

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